Quantitative data analysis and multiparameter inversion of Biot wave data: insights from topology and geometry

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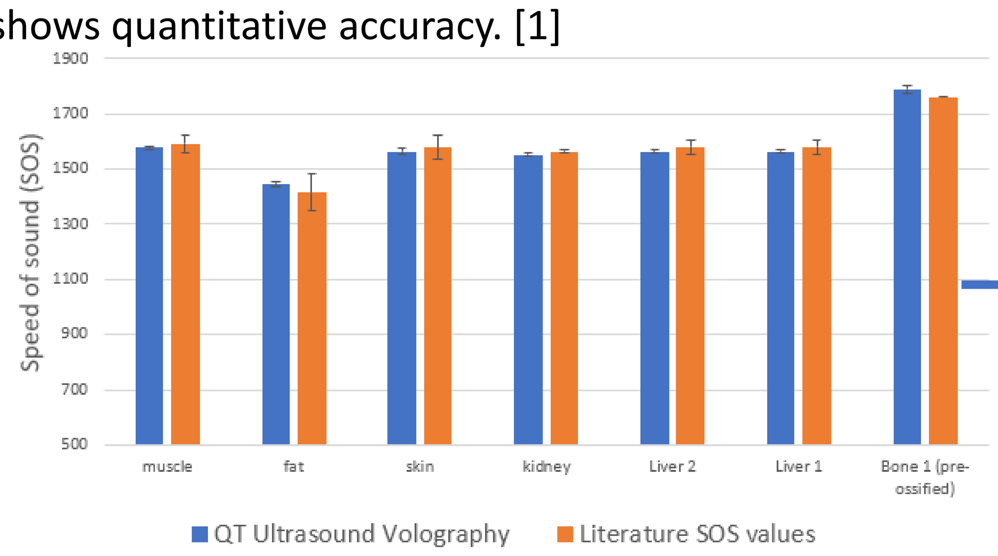
Introduction

3D ultrasound tomography (3D-UT) is employed in breast imaging presently and has recently been extended to orthopedic applications as a proof of concept (POC) with a cadaver knee.

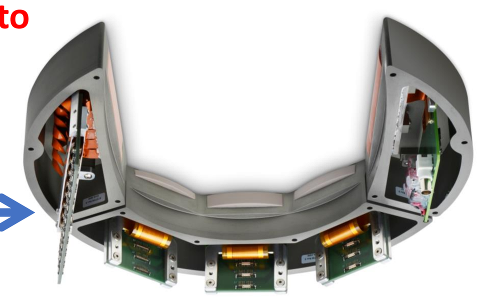
Speed of sound reconstruction shows quantitative accuracy. [1]



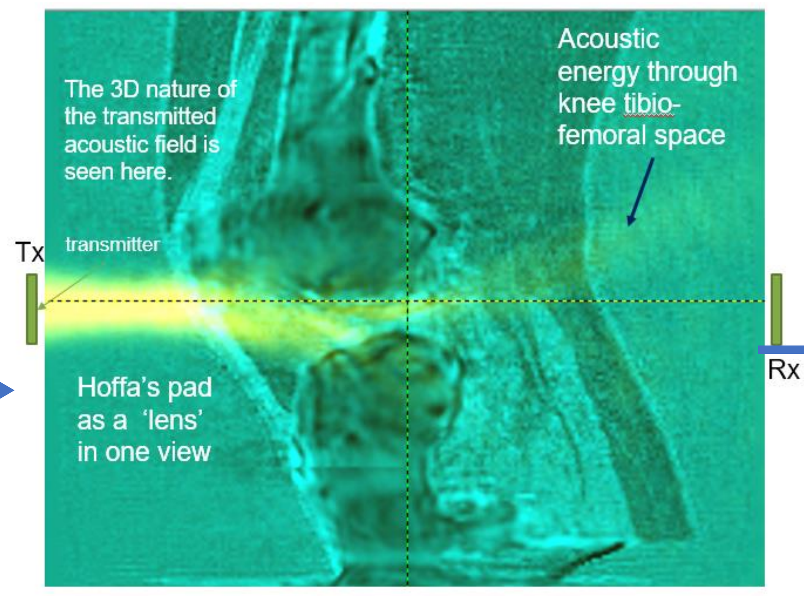
Quantitative accuracy has been verified for soft tissue [1]

Methods

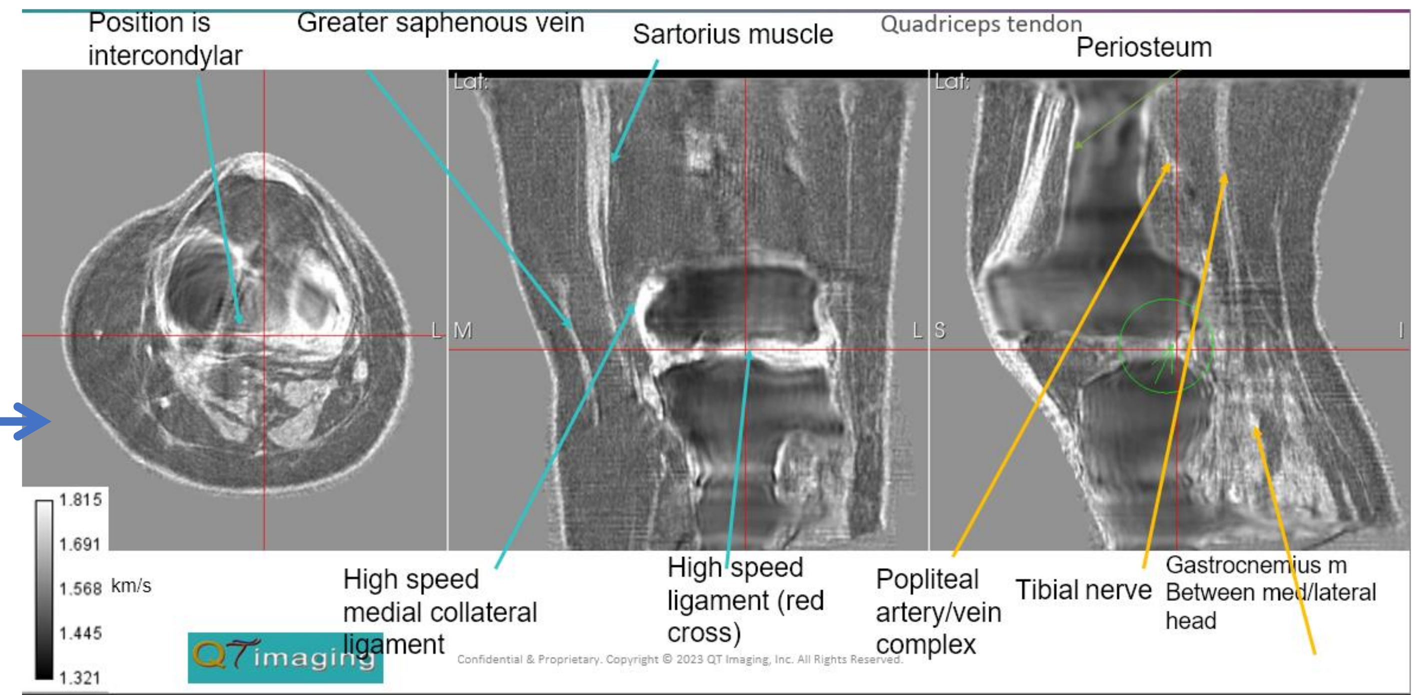
Read left to right



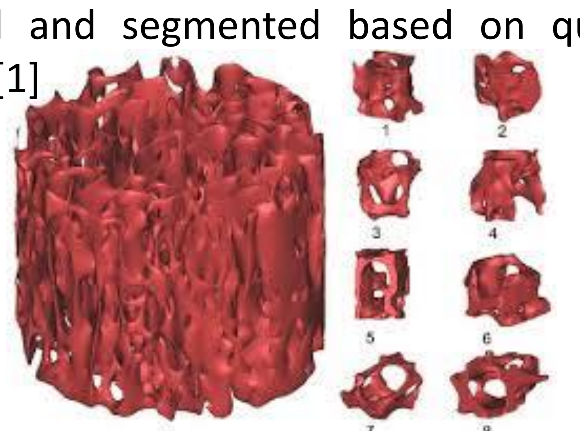
QT Imaging Scanner with water bath and 8 by 256 column array and independent transmitter array – cadaver knee imaged and segmented based on quantitative speed of sound [1]



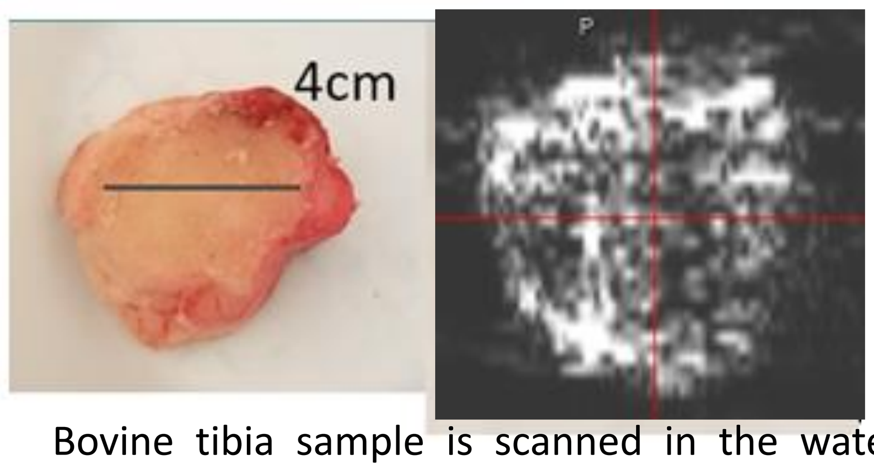
3D nature of ultrasonic wave requires 3D full wave inversion imaging algorithm



Quantitative accuracy of ligaments, muscle, cartilage, tendons, fat, skin, etc. However, bone interior appears slower than expected.



Femur and Tibia bone are cancellous with bone matrix filled with marrow – accurately modelled with Biot-Johnson theory

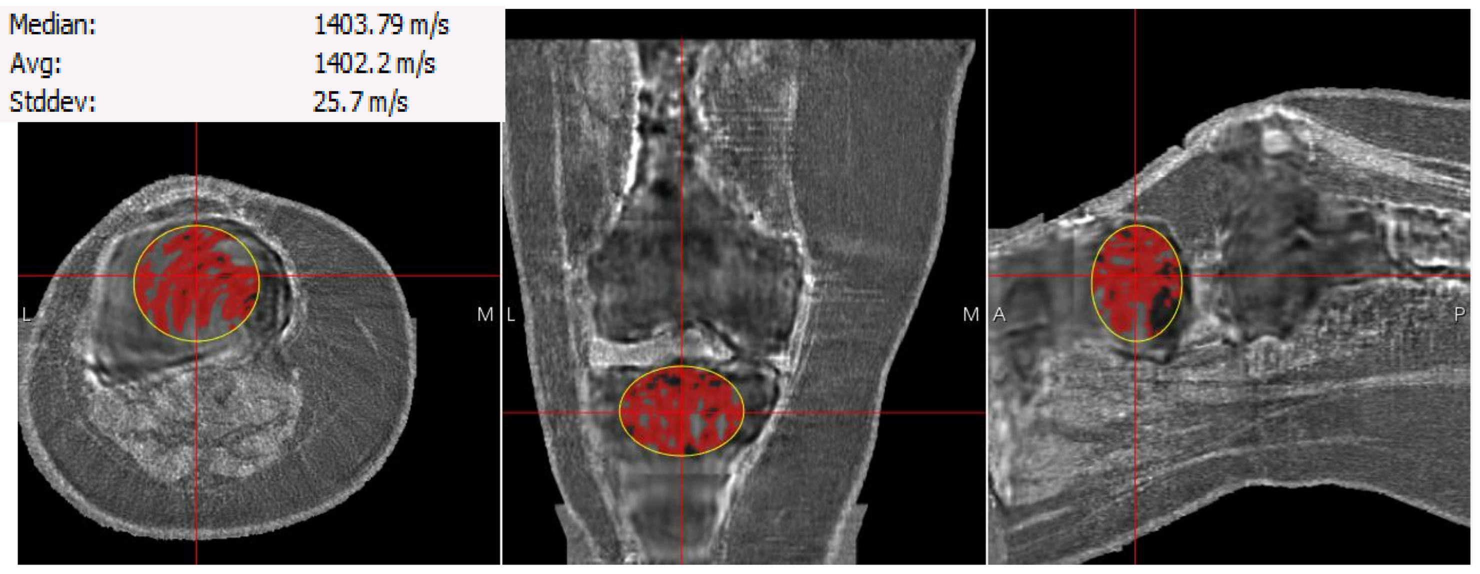


Bovine tibia sample is scanned in the water bath scanner – attenuation image on right

$$\begin{pmatrix} \hat{\rho}_{11} & \hat{\rho}_{12} \\ \hat{\rho}_{21} & \hat{\rho}_{22} \end{pmatrix} \begin{pmatrix} \partial_t^2 \mathbf{u} \\ \partial_t^2 \mathbf{U} \end{pmatrix} = \begin{pmatrix} P & Q \\ Q & R \end{pmatrix} \begin{pmatrix} \nabla(\nabla.\mathbf{u}) \\ \nabla(\nabla.\mathbf{U}) \end{pmatrix} - \begin{pmatrix} N(\nabla \times \nabla \times \mathbf{u}) \\ 0 \end{pmatrix}$$

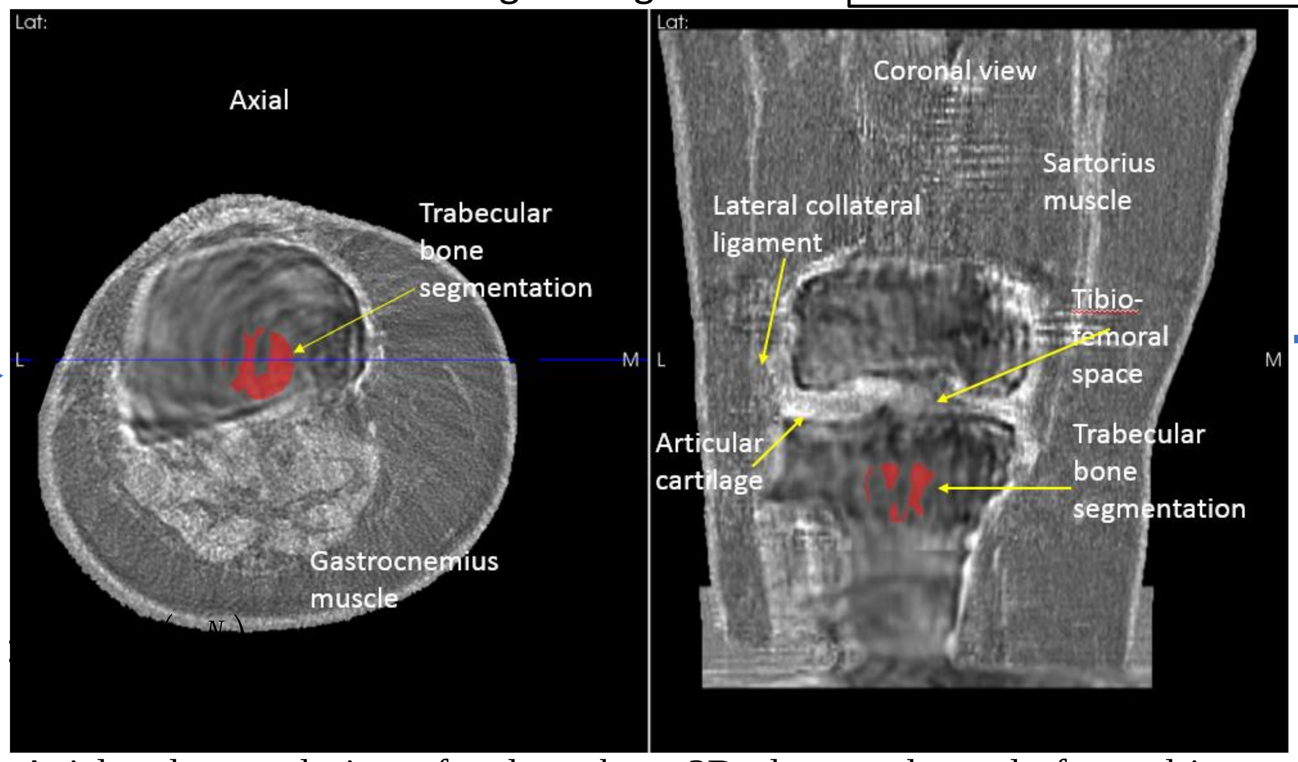
Biot theory: $\hat{\rho}_{ij}$ *N, Q, R, and P* coefficients relate to mass density of fluid, dynamic tortuosity, permeability and viscous characteristic length and arise out of the contribution of Johnson et al.[2] and Biot's original paper. This theory predicts a 'slow' compressional wave: SOS =~1470 m/s (temperature and frequency dependent). Which is slower than the standard compressional wave.

Results

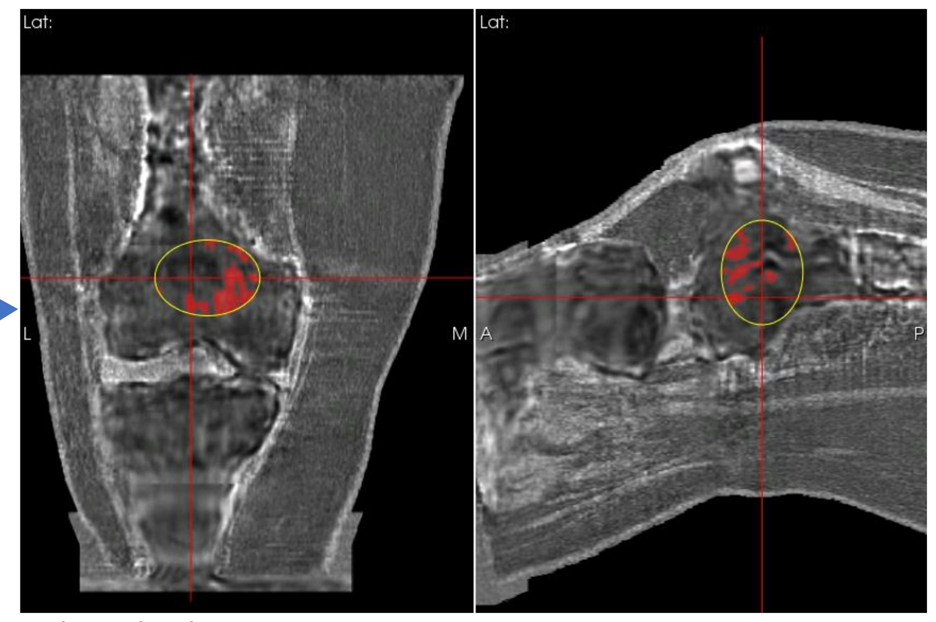


Segmentation of marrow in tibia based on the SOS and constrained by ellipsoid gives average value 1402.2 m/s

Coronal view of SOS showing the marrow segmentation in tibia
SOS for this segmentation is 1389 (σ = 23.5) m/s at 1.3 MHz and 33 C

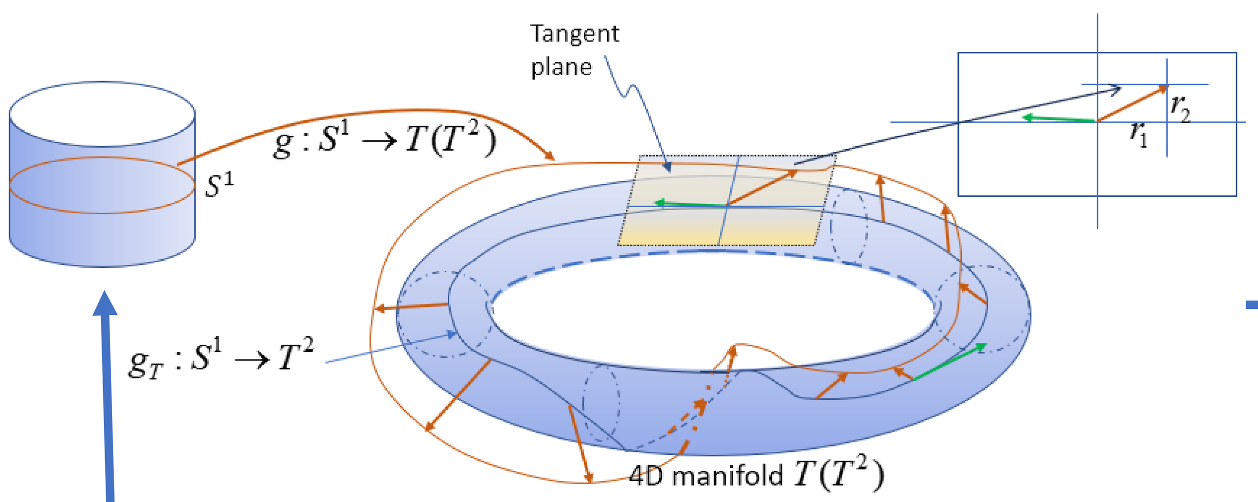


Axial and coronal view of cadaver knee 3D ultrasound speed of sound image (SOS) with bone segmented region shown in red has an average SOS = 1466 m/s (SD = 29.9) which agrees with estimates of slow P wave SOS in human femur at 0.4 – 0.8 MHz



Trabecular bone segmentation in Femur

Median:	1466.29 m/s
Avg:	1472.5 m/s
Stddev:	25.1 m/s

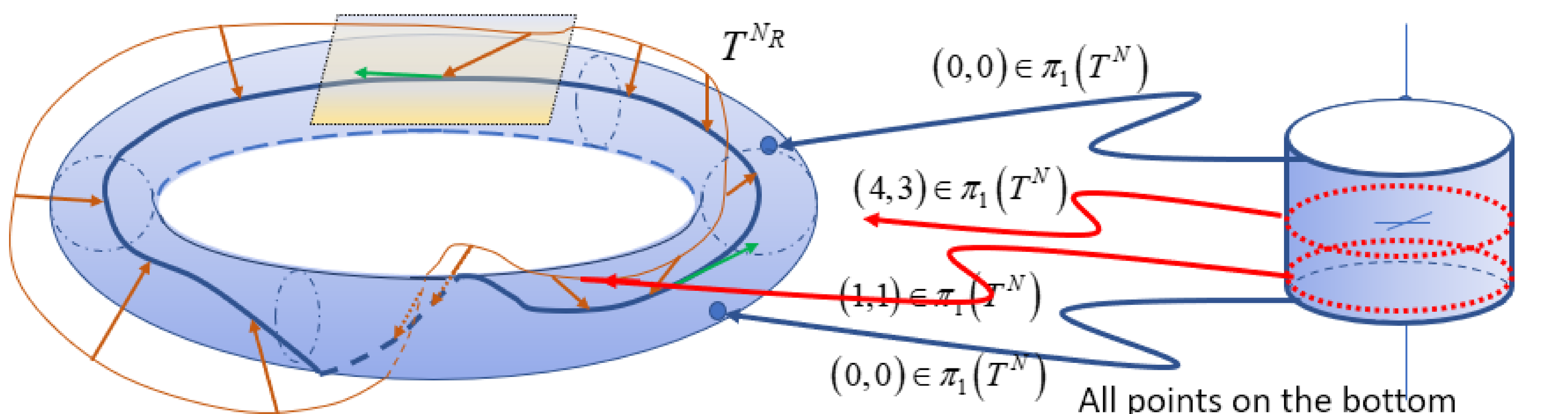


The map at fixed level maps circle to *N complex numbers interpreted as* tangent bundle to the *N-torus* T^N

$$\theta \in S^1 \to (\theta_1, \cdots, \theta_N, r_1, \cdots, r_N) \in T^N \times R^N \approx T(T^N)$$
$$g: S^1 \to T(T^N)$$

Idealized water tank as cylinder gives angle and level $(\theta, l) \in S^1 \times R$

The red arrows above show this is homotopic to a map to the torus-green line $g_T: S^1 \to T^N$
However, this is by definition the first homotopy group of the *N-torus,* $\pi_1(T^N)$



Intermediate maps are topologically distinct

All points on the bottom boundary MAY get mapped to the same point in torus T^{N_R}

It is known that $\pi_1(T^N) \approx Z \otimes \cdots Z \otimes Z$

A typical element of $\pi_1(T^N) \approx Z \otimes \cdots Z \otimes Z$ has the form $\mathbf{N} \equiv (n_1, \cdots, n_N) \in \pi_1(T^N)$, that is, it is an *n-tuple of integers indicating* the number times the curve winds around the torus in the orthogonal directions.
The case for *N=2* is shown above for clarity. Note these represent the homotopically distinct maps $g_T: S^1 \to T^N$, and thus the possibility of phase wrapping.
The vertical steps must be small enough to ensure that these integral values only change by unity: $n_j \to n_j \pm 1$
The vertical direction can also represent the frequencies (.4, .5, … , 1.3 MHz) in which case the construction indicates that small steps must be made in frequency so that any change in integral values is by one value only.

	Literature values	QTUS measured values
human bone marrow	~1410 m/s	1402, 1389
Biot slow wave SOS	~1470 m/s [2, 3]	1472, 1466 m/s

The table above shows the close correspondence of our measured values for the Biot slow compressional wave compared to the literature values [2,3,4]. There is some discrepancy due to temperature and frequency differences. The Biot slow wave value is related to porosity. This phenomenological relationship may help in detecting osteoporosis.

Low Resource Environments (LRE): the scanner is self-contained (120V) and can be transported easily on a mobile platform to LRE's and historically underserved populations.

References

[1] J. Wiskin, B. Malik, C. Ruoff, N. Pirshafiey, M. Lenox, and J. Klock, "Whole-Body Imaging Using Low Frequency Transmission Ultrasound," *Academic Radiology*, 2023/02/24/ 2023.

[2] D. L. Johnson, J. Koplik, and R. Dashen, "Theory of dynamic permeability and tortuosity in fluid-saturated porous media," *Journal of Fluid Mechanics*, vol. 176, pp. 379-402, 2006

[3] A. Hosokawa and T. Otani, "Ultrasonic wave propagation in bovine cancellous bone," *J Acoust Soc Am*, vol. 101, pp. 558-62, Jan 1997.

[4] K. Wear, Y. Nagatani, K. Mizuno, and M. Matsukawa, "Fast and slow wave detection in bovine cancellous bone in vitro using bandlimited deconvolution and Prony's method," *J Acoust Soc Am*, vol. 136, pp. 2015-24, Oct 2014.

Conclusion

The reconstruction interior to the bone appears to reconstruct trabecula with slow wave SOS and marrow with accepted literature values for this temperature and frequency. The slow (Biot) wave is known to dominate the fast P wave, supporting a valid reconstruction with this wave interior to trabecular bone. This may have clinical applications since a reconstruction using the full Biot model may give effective parameters that correlate to osteoporosis.

The fact that these values are close to the literature values for marrow and the slow Biot wave are supportive but not conclusive at this time. A full inversion of the equations in (1) would give further justification that we are seeing a true slow Biot wave.

The representation of the DA as a subset of the first homotopy group of the N-torus $\pi_1(T^N) \approx Z \otimes \cdots Z \otimes Z$, provides a natural context for understanding phase unwrapping from a manifold perspective, and gives natural constraints on frequencies and distance between DA levels.

Acknowledgements

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